

Eg: Let $(G, *) = (\mathbb{R}^2, +)$ be a gp.
and $H = \{(x, 0) : x \in \mathbb{R}\} < G$

Solu:

$$\begin{aligned}(1, 2) * H &= \{(1, 2) + h : h \in H\} \\&= \{(1, 2) + (x, 0) : x \in \mathbb{R}\} \\&= \{(x+1, 2) : x \in \mathbb{R}\} \\&= \{(y, 2) : y \in \mathbb{R}\} \\(1, 2) * H &= \{(x, 2) : x \in \mathbb{R}\} \\&= H * (1, 2)\end{aligned}$$

QUOTIENT SPACE:

Let $V(F)$ be a v.s. and W be a subsp. of V .
Then, define $\frac{V}{W} = \{a+W : a \in V\}$
 $\downarrow$
int. op.
 $=$ set of all possible cosets of W

★ $(\frac{V}{W}, +, \cdot)$ is a v.s. with the operations
 $+ : \frac{V}{W} \times \frac{V}{W} \rightarrow \frac{V}{W}$ s.t.

$$\begin{aligned}+ : (a+W, b+W) &= (a+W) + (b+W) \\&= (a+b)+W\end{aligned}$$

$$\cdot : F \times \frac{V}{W} \rightarrow \frac{V}{W} \text{ s.t. } \alpha \cdot (a+W) = \alpha \cdot a + W.$$

$(\frac{V}{W}, +, \cdot)$ is known as v.s. $\Rightarrow$ QUOTIENT SPACE
of V related to W .

- zero element of $\frac{V}{W}$ is $0+W=W$
- inverse element of $a+W \in \frac{V}{W}$ is $-a+W$